

LPV Control of Full-Vehicle Vertical Dynamics using Semi-Active Dampers

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Abstract: Semi-active suspensions enable a mitigation of the design conflict between ride comfort and road-holding by allowing the continuous control of the damper force. During the design of control laws for semi-active dampers, the inherent actuator limits constitute a major performance restriction. Accordingly, these constraints should be explicitly considered during controller design. In this work, the actuator constraints are addressed by LPV control methods which offer a rigorous framework to incorporate them as so-called saturation indicator parameters in the LPV plant. Additionally, saturation indicator dependent weighting filters are employed in a mixed sensitivity scheme in order to honor stability over performance in the event of saturation. Furthermore, the LPV controller is synthesized using a full-vehicle model, which enables the individual tuning of the heave, pitch and roll motion. The full-vehicle model, however, leads to a significant increase in complexity of the controller synthesis problem compared to a quarter-vehicle model. Therefore, a disturbance-feedforward output-feedback controller design is adopted to exploit the separation principle. The performance of the proposed full-vehicle LPV controller is compared to two quarter-vehicle LPV controllers, a Skyhook/Groundhook controller and passive suspension configurations in an extensive simulation investigation. The results illustrate the improved pareto optimum of the LPV full-vehicle controller and the suitability of the proposed design scheme.

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1. INTRODUCTION

The suspension system of a vehicle significantly contributes to road-holding and ride comfort by ensuring a sufficient tire-road contact and a good isolation of the vehicle body from road excitations, respectively. Generally, design of suspension systems means to find a compromise between these two conflicting demands since optimal ride comfort and optimal road-holding have different requirements. For example, optimal ride comfort and road-holding in a passive suspension require different damping coefficients around the body and wheel resonance frequencies, see Mitschke and Wallentowitz (2004). Fig. 1 schematically illustrates this design conflict, e. g. the dotted black lines represent the variation of the damping coefficient d with a fixed body spring stiffness. Compared to passive suspension systems, semi-active ones enable a mitigation of the above described design conflict by allowing the continuous control of the damper force within the force limits. For a passive suspension, the solid black line represents the optimal trade-off (pareto-front) between ride comfort and road-holding. The dashed green line shows the pareto-front of the corresponding semi-active suspension. A detailed investigation on pareto-fronts of

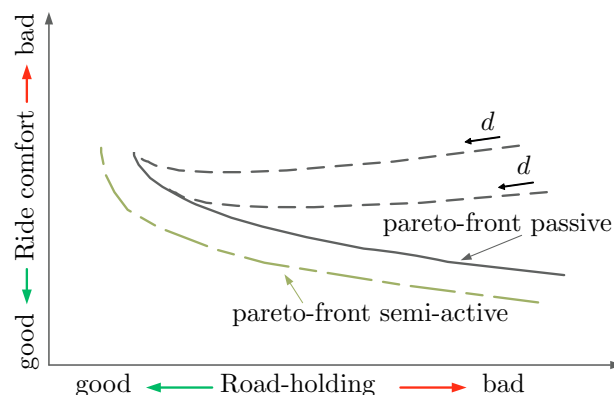


Fig. 1. Schematic illustration of the suspension design conflict (ride comfort vs. road-holding)

passive, semi-active and active suspension systems can be found in Tseng and Hrovat (2015).

The inherent actuator force limits of semi-active dampers are one major restriction which has to be handled during controller design. In literature, several control approaches such as Skyhook (Karnopp et al. (1974)), Groundhook (Valášek et al. (1997)), acceleration driven damping (Savaresi and Spelta (2007)) and clipped control (Tseng and Hrovat (2015)) are investigated, which all implicitly handle the actuator constraints. A comparative study on

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Fig. 2. left ROMO; right wheel robot unit with traction motor, steering system, brake and semi-active damper

a quarter-vehicle model of these control approaches can be found in Poussot-Vassal et al. (2012). Recently control approaches such as model-predictive control (Canale et al. (2006); Giorgetti et al. (2006)), adaptive anti-windup (Ahmed and Svaricek (2014)) and linear parameter varying (LPV) control (Poussot-Vassal et al. (2008); Do et al. (2010); Nguyen et al. (2015)) which explicitly consider the actuator constraints, have been investigated.

The two latter approaches adapt the general LPV anti-windup control method proposed in Wu et al. (2000) to the case of semi-active suspensions. In particular, this general LPV method features guaranteed stability and graceful performance degradation in the event of actuator saturation. This is achieved by directly modeling the actuator constraints in the LPV synthesis model with scheduling parameters called saturation indicators. Based on Wu et al. (2000), the authors in Do et al. (2010) present the design of a polytopic LPV controller with a quarter-vehicle synthesis model. This model employs two scheduling parameters to account for the nonlinearity of the adapted Shuqi Guo damper model and for the actuator limits of the semi-active damper. In order to realize optimal ride comfort and road-holding, Do et al. (2012) introduce a pareto optimization of the weighting filters employed in the LPV controller design.

In this work, a full-vehicle LPV controller for a semi-active suspension is developed. Similarly to Do et al. (2010), the LPV controller design is derived from Wu et al. (2000). In comparison to the constant weighting scheme in Do et al. (2010), however, a new saturation indicator dependent weighting scheme is introduced. Additionally, the output-feedback controller design follows a disturbance-feedforward scheme, see Doyle et al. (1989). Within such a design scheme the separation principle can be exploited with guaranteed closed-loop performance and stability. In this way, the complexity of the LPV controller synthesis and code execution on real-time hardware can be significantly reduced. The complexity of the LPV controller is also considered for the selection of the performance outputs and their weighting filters leading to a different choice of performance outputs and weighting filters compared to Do et al. (2010). Furthermore, in a full-vehicle controller design, the heave, pitch and roll motion of the vehicle body can be individually weighted and accordingly an improved compromise between ride comfort and road-holding can be expected.

The controller development is performed for the RObo-Mobil (ROMO, see Brembeck et al. (2011), Bunte et al. (2014), and Fig. 2 left) which is an innovative robotic electric research-platform at the Robotics and Mechatronics Center (RMC) of the German Aerospace Center (DLR). It is equipped with four semi-active dampers as described

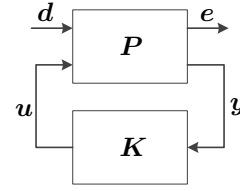


Fig. 3. Interconnection of generalized plant P with controller K

in Fleps-Dezasse and Tobolar (2015). Each wheel unit (see Fig. 2 right) integrates a traction motor, a steering system and a brake in the wheel carrier. This leads to a body mass to wheel mass ratio smaller than 4 which tightens the design conflict between ride comfort and road-holding.

2. LPV CONTROL WITH ACTUATOR CONSTRAINTS

The anti-windup LPV controller design proposed by Wu et al. (2000) offers a general framework to realize MIMO controllers, which account for actuator saturation. The approach utilizes saturation indicators to directly schedule the controller according to the saturation status. An open-loop LPV description G of an LTI plant with actuator constraint is given by:

$$\begin{bmatrix} \dot{x} \\ y \end{bmatrix} = \begin{bmatrix} A & B\Theta \\ C & D\Theta \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix}, \quad (1)$$

where $x \in \mathbb{R}^{n_x}$, $u \in \mathbb{R}^{n_u}$, $y \in \mathbb{R}^{n_y}$ and the time-varying saturation indicator matrix $\Theta \in \mathbb{R}^{n_u \times n_u}$. In this work, the actuator constraints are assumed to be decoupled and state-dependent. Due to the former property, the saturation matrix Θ takes a diagonal form $\Theta = \text{diag}(\theta)$ with the saturation indicator vector $\theta = [\theta_1 \ \theta_2 \ \dots \ \theta_{n_u}]^T$. The latter property leads to a limited control input $\sigma(u_i)$ of the form

$$\sigma(u_i) = \begin{cases} u_i & u_i^{\min}(x) < u_i < u_i^{\max}(x) \\ u_i^{\min}(x) & u_i^{\min}(x) > u_i \\ u_i^{\max}(x) & u_i^{\max}(x) < u_i \end{cases} \quad (2)$$

with the number of actuators $i = 1, 2, \dots, n_u$. The saturation indicator θ_i of the i -th actuator can then be defined as $\theta_i = \frac{\sigma(u_i)}{u_i}$ with $u_i^{\min}(x) < 0$ and $u_i^{\max}(x) > 0$, and thus $\theta_i \in (0, 1]$. A parameter value $\theta_i = 1$ describes the situation when the actuator i is not saturated and a parameter value $\theta_i < 1$ reflects the amount of actuator saturation. The saturation indicators θ are assumed to continuously evolve over time, to be measurable or estimable in real-time, and to be limited in range to the compact set $\mathcal{P}$.

The proposed LPV controller design uses the generalized plant concept (Skogestad and Postlethwaite (2005)). Fig. 3 shows the interconnection of the generalized plant P with the controller K and the corresponding measurement output y , control input u , disturbance input d and performance output e . The generalized plant P is obtained from the open-loop plant G firstly by extending G with disturbance inputs and performance outputs and secondly by introducing suitable weighing filters. In state-space description P can be expressed as

$$\begin{bmatrix} \dot{x} \\ e \\ y \end{bmatrix} = \begin{bmatrix} A & B_1 & B_2\Theta \\ C_1 & D_{11} & D_{12}\Theta \\ C_2 & D_{21} & D_{22}\Theta \end{bmatrix} \begin{bmatrix} x \\ d \\ u \end{bmatrix} \quad (3)$$

The LPV controller synthesis is formulated as an optimization problem seeking the controller $\mathbf{K}$, that minimizes the induced L_2 -norm from $\mathbf{d}$ to $\mathbf{e}$ of the closed-loop system, while simultaneously ensuring its stability.

$$\min_{\mathbf{K}, \gamma} \gamma \quad \text{s. t.} \quad \begin{cases} \mathcal{F}_l(\mathbf{P}, \mathbf{K}) \text{ is internally stable} \\ \|\mathcal{F}_l(\mathbf{P}, \mathbf{K})\|_{i2} < \gamma \end{cases} \quad (4)$$

In eq. (4), γ represents the least upper bound of the induced L_2 -norm of the closed-loop system denoted by the lower fractional transformation $\mathcal{F}_l(\mathbf{P}, \mathbf{K})$. The resulting controller $\mathbf{K}$ has the form (Wu et al. (2000))

$$\begin{bmatrix} \dot{\mathbf{x}}_k \\ \mathbf{u} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_k & \mathbf{B}_k \\ \mathbf{\Theta}^{-1}\mathbf{C}_k & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x}_k \\ \mathbf{y} \end{bmatrix} \quad (5)$$

The optimization problem in (4) can be converted into a semi-definite program (SDP), which can be efficiently solved. For details on the derivation of the corresponding parameter dependent linear matrix inequalities (LMI) and a proof of the existence of a solution of the optimization problem, the reader is referred to Apkarian et al. (1995) and Wu (1995). In this work, a gridding approach with a parameter-independent Lyapunov function is used to cast the infinite dimensional optimization problem into a finite dimensional one. The controller synthesis itself is carried out in Matlab using the LPVTools toolbox (Hjartarson et al. (2013)).

3. DISTURBANCE-FEEDFORWARD DESIGN

The design of an output-feedback LPV controller in a disturbance-feedforward (DF) scheme reduces the computational complexity of the controller synthesis as the controller design can be divided into two steps. First an appropriate state-observer gain $\mathbf{L}$ has to be found and second the DF output-feedback controller is obtained from the solution $\mathbf{F}$ of a state-feedback problem, see Doyle et al. (1989); Saupé and Pfifer (2011). The advantage of the DF scheme compared to the separate design of an observer and a state-feedback controller arises from the guaranteed closed-loop performance and stability when interconnecting $\mathbf{P}_{DF}$ with $\mathbf{K}_{DF}$. Compared to a classical output-feedback controller synthesis, the advantages are a significant reduction of the problem complexity and an easier implementation on real-time hardware. The disadvantages of the DF scheme, however, are restrictive structural requirements imposed on the generalized plant:

- a) $(\mathbf{A}, \mathbf{B}_2\mathbf{\Theta})$ is parameter-dependent stabilizable;
- b) $\text{rank}(\mathbf{D}_{12}\mathbf{\Theta}) = \dim(\mathbf{u})$;
- c) $\mathbf{D}_{11} = \mathbf{0}$, $\mathbf{D}_{22} = \mathbf{0}$ and $\mathbf{D}_{21} = \mathbf{I}$; and
- d) $\mathbf{A} - \mathbf{B}_1\mathbf{C}_2$ is parameter-dependent stable

The assumptions a) and b) are common in LPV control design, see Wu (1995), while the assumption c) and d) are specific for a DF problem. The assumption d) can be met by choosing $\mathbf{B}_1 = \mathbf{L}$ as described in Doyle et al. (1989). The assumption $\mathbf{D}_{21} = \mathbf{I}$ restricts the disturbance $\mathbf{d}$ to directly act on the measurements $\mathbf{y}$ and to disturb the state derivatives through the observer gain $\mathbf{L}$. From the solution $\mathbf{F}(\boldsymbol{\theta})$ to the state-feedback problem, the DF output-feedback controller $\mathbf{K}_{DF}$ can be constructed as

$$\mathbf{A}_k(\boldsymbol{\theta}) = \mathbf{A} + \mathbf{B}_2\mathbf{\Theta}\mathbf{F}(\boldsymbol{\theta}) - \mathbf{L}\mathbf{C}_2. \quad (6)$$

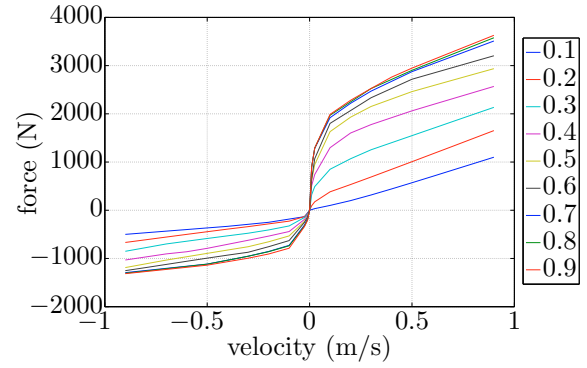


Fig. 4. Speed-force diagram of semi-active damper with normalized control signal u_d

4. CONTROL-ORIENTED VEHICLE MODEL

An important step during model-based controller design consists in developing a suitable plant model. In this paper, the vertical dynamics of ROMO is approximated by a linear full-vehicle model (FVM) with seven degrees-of-freedom (DoF). In comparison to quarter-vehicle models (QVM), commonly used for semi-active damper control, a FVM models the body heave, roll and pitch motion. Accordingly, one central controller for the vertical dynamics of the whole vehicle can be synthesized, instead of four separate QVM controllers. The equations of motion corresponding to the seven DoF of the FVM can be found in Savaresi et al. (2010); Unger et al. (2013); and Fleps-Dezasse and Tobolar (2015). For the FVM, the body spring and the tire stiffness and damping are assumed linear. The semi-active damper force is split into two parts: a linear part with the damping coefficient d_0 and a virtual control force u_{F_d} , as in Poussot-Vassal et al. (2008). In this way, the passivity constraints of the actual damper force is circumvented and the virtual constraints can be described by $\mathbf{\Theta}$. The dynamics of u_{F_d} are modeled as the first-order linear system

$$\dot{x}_{F_d} = 2\pi\omega(u_{F_d} - x_{F_d}) \quad (7)$$

with the damper force x_{F_d} and the bandwidth ω . Additionally, a simple model of road disturbances as in Unger et al. (2013) is incorporated. This road model approximates the road profile height x_g of a stochastic road by the first-order filter

$$\dot{x}_g = -\beta v_{ref}x_g + v_{ref}d_g. \quad (8)$$

The parameters $\beta = 0.2\text{rad/m}$ and $v_{ref} = 30\text{m/s}$ are taken from Unger et al. (2013). The disturbance input d_g is introduced in reference to the observer synthesis in section 5.1. The resulting state vector of the FVM comprises 22 elements, namely the position and velocity of the wheels, and body heave, roll and pitch. Furthermore, the four damper forces, and four road profile heights are part of the FVM state.

The speed-force diagram of the electro-hydraulic semi-active dampers of ROMO is shown in Fig. 4. The damper features a highly unsymmetrical force characteristic with reduced compression forces. This property improves ride comfort during obstacle events like bumps and potholes, especially for passive suspension configurations. Due to the nonlinear characteristic of the damper an inverse damper

model is introduced into the control loop, see Fig. 5. This inverse model performs a transformation and linearization of the damper input such that the controller can directly command a damper force, see Unger et al. (2013) for details.

The ROMO is equipped with seven accelerometers measuring the wheel accelerations $\ddot{x}_w$ and the body acceleration $\ddot{x}_{bs}$ at three locations distributed in the body's horizontal plane. Additionally, four potentiometers are employed to measure the damper deflection. For the observer design, however, the calculated damper velocity is used as the velocity signal is not affected by offsets, e. g. due to the expected static friction in ROMO's suspension or due to load variations.

In the control structure in Fig. 5, the observer takes the saturated damper forces $F_{d,sat}$ and the measurements y as inputs and outputs the estimated plant state $\hat{x}$. Subsequently, the controller calculates the damper forces $F_d = d_0 v_d + u_{F_d}$ with v_d the damper velocities using the saturation indicators θ and finally the inverse damper model transforms the controller output F_d into the damper input u_d .

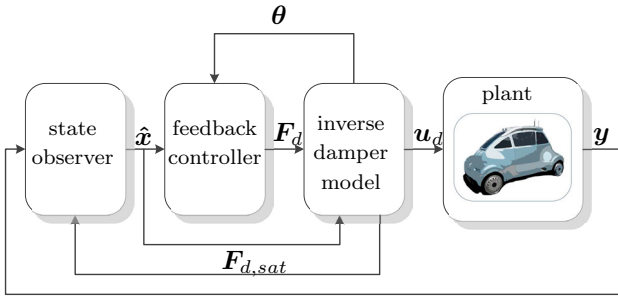


Fig. 5. Control structure of the DF output-feedback controller

5. MIXED SENSITIVITY DESIGN WITH ACTUATOR CONSTRAINTS

The structure of the DF output-feedback controller with inverse damper model is depicted in Fig. 5. After the input transformation as described in section 4, it is assumed that the vertical dynamics of the ROMO can be described by a linear vehicle model with input constraints and consequently, no further scheduling parameters besides θ have to be considered in controller design.

5.1 Observer Design

The first step of a DF output-feedback controller design consists of finding a suitable observer gain. This step simplifies to a classical $\mathcal{H}_\infty$ state-observer design problem, due to the assumed linear vehicle model. Two types of disturbances are incorporated in the observer synthesis:

- stochastic road disturbances for each wheel d_g all weighted by the scalar w_g ,
- and sensor noise disturbances d_n weighted by the diagonal matrix W_n .

The performance output is defined as the error between the actual plant state x and the estimated plant state $\hat{x}$

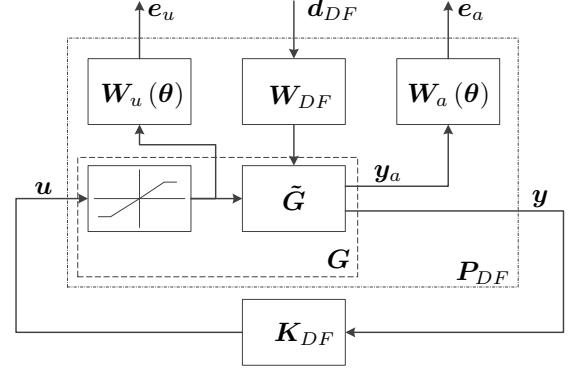


Fig. 6. DF mixed sensitivity S/KS scheme

and weighted by the diagonal matrix $W_{\Delta e_x}$. Accordingly, the generalized plant for observer design takes the form:

$$\begin{bmatrix} \dot{x} \\ \Delta e_x \\ y \end{bmatrix} = \begin{bmatrix} A & B_{oe} & 0 \\ W_{\Delta e_x} & 0 & -W_{\Delta e_x} \\ C_2 & D_{oe} & 0 \end{bmatrix} \begin{bmatrix} x \\ d_{oe} \\ \hat{x} \end{bmatrix} \quad (9)$$

with the new input and feedthrough matrices B_{oe} and D_{oe} . The observer gain L can be now computed using (9) by solving the $\mathcal{H}_\infty$ state-observer design problem.

5.2 DF Output-Feedback Controller Design

The controller is synthesized in a mixed sensitivity S/KS weighting scheme with disturbance input d_{DF} , performance output e_a , and control effort e_u as illustrated in Fig. 6. In this figure, $\tilde{G}$ denotes the open-loop plant without actuator saturation. The disturbance input d_{DF} and the corresponding filter W_{DF} must meet the structural requirements introduced in section 3. This means, the road disturbance described in (8) cannot be utilized in a DF controller design. The disturbance d_{DF} , however, can be interpreted as a more general disturbance with road disturbances, load disturbances and sensor noise disturbances as special realizations.

In case of semi-active damper control, the aim is only disturbance rejection, thus no reference input is incorporated in the weighting scheme and the performance objective with the parameter-dependent weighting filters $W_a(\theta)$ and $W_u(\theta)$ can be summarized as

$$\|\mathcal{F}_l(P, K)\|_{i2} = \left\| \frac{W_a(\theta) S_{DF,a} W_{DF}}{W_u(\theta) K S G_{DF,y} W_{DF}} \right\|_{i2} < \gamma \quad (10)$$

with the sensitivity function $S = (I - G_y K)^{-1}$ and the transfer function $S_{DF,a} = G_a K S G_{DF,y} + G_{DF,a}$. In (10), the lower index DF denotes transfer function with the input d_{DF} . Similarly y and a indicate transfer functions with the measurement output y and the performance output y_a , respectively.

The selection of performance outputs and corresponding weighting filters considers the number of extra states introduced into the generalized plant as the number of states is directly related to the computational effort during controller synthesis and code execution, with the former scaling particularly badly. Hence, the body heave, roll and pitch velocities, as well as the wheel velocities, are selected as performance outputs because a velocity output

rolls-off one decade faster than an acceleration output. Accordingly, the order of the employed weighting filters can be reduced. The body related outputs are all weighted by first-order low-pass filters and the wheel related outputs are weighted by simple scalar gains. This weighting scheme yields significantly fewer extra states, compare to the weighting schemes proposed in Poussot-Vassal et al. (2008) and Do et al. (2010), which employ acceleration and force signals with second-order weighting filters. The drawback of the proposed weighing scheme may be a decrease in performance as the formulated controller design problem does not directly minimize the design targets ride comfort and road-holding, which are derived from body accelerations and dynamic wheel loads, respectively.

5.3 Parameter-Dependent Weights Selection

Obviously, the LPV controller in (5) will increase its control action in the event of saturation through Θ^{-1} . Thus, the controller synthesis problem requires parameter-dependent weighting filters in order to realize a successful controller design (Wu et al. (2000)). In the following, the desired parameter-dependence of $\mathbf{W}_a$, comprised of the body related filters $\mathbf{W}_{a,b}$ and the wheel related filters $\mathbf{W}_{a,w}$, and of $\mathbf{W}_u$ is introduced for scalar inputs and outputs. For this case, the induced L_2 -norm in (10) simplifies to an Euclidean vector norm of the form (Skogestad and Postlethwaite (2005))

$$\sup \left(\sqrt{|W_a S_{DF,a} W_{DF}|^2 + |W_u K S_{G_{DF,y}} W_{DF}|^2} \right) < \gamma. \quad (11)$$

Eq. (11) implies that the least upper bound of the individual elements is smaller than γ and thus

$$\begin{aligned} \|S_{DF,a} W_{DF}\|_{i_2} &< \gamma W_a^{-1}(\theta), \\ \|K S_{G_{DF,y}} W_{DF}\|_{i_2} &< \gamma W_u^{-1}(\theta). \end{aligned} \quad (12)$$

The desired dependence of the weighting filters W_a and W_u on θ in the event of saturation can now be summarized as:

- increase $W_u(\theta)$ to reduce the controller output,
- reduce $W_a(\theta)$ to honor stability over performance.

Fig. 7 illustrates such a θ -dependence of W_u and W_a . As introduced at the beginning of this section, the weighting filter W_a related to the vehicle body velocity is realized as a first-order low-pass filter with θ -dependent gain. The actuator related filter W_u is chosen as a static θ -dependent gain.

The θ -dependence introduced for scalars d , e_a and e_u is now extended to the general case of vectorial inputs and outputs. In a full-vehicle application, the four semi-active dampers offer redundancy with respect to influencing the vehicle body motion. Therefore, the performance requirements should be gradually reduced according to the saturation status of the dampers. To realize this, the θ -dependence of the weighting filter $\mathbf{W}_{a,b}$ is defined as

$$\mathbf{W}_{a,b}(\theta) = \left(\frac{1}{n_u} \sum_{i=1}^{n_u} \theta_i \right) \begin{bmatrix} w_{b,h} & 0 & 0 \\ 0 & w_{b,r} & 0 \\ 0 & 0 & w_{b,p} \end{bmatrix} \Psi_{a,b} \quad (13)$$

with the diagonal weighting filter matrix $\Psi_{a,b}$ consisting of three first-order low-pass filters with magnitude one, and the heave, roll and pitch gains $w_{b,h}$, $w_{b,r}$ and $w_{b,p}$. The

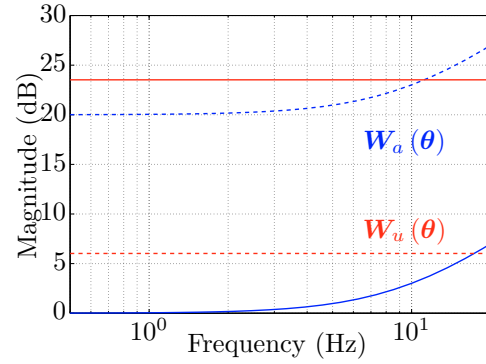


Fig. 7. Saturation indicator dependence of weighting filters $W_{a,b}$ and W_u (solid - unsaturated; dashed - saturated)

decrease in gain of the performance weight $\mathbf{W}_{a,b}$ due to one strongly saturated actuator ($\theta_i \rightarrow 0$) is equal to $\frac{1}{n_u}$, while $\mathbf{W}_{a,b}$ approaches a minimum value close to zero if all actuators are strongly saturated.

The wheel velocity weighting $\mathbf{W}_{a,w}$ is defined as

$$\mathbf{W}_{a,w}(\theta) = w_w \Theta \quad (14)$$

with the gain w_w . As the wheel motion can only be influenced by the attached damper, the diagonal saturation matrix Θ can be employed to individually reduce the wheel performance demand.

In this work, a θ -dependent weighting filter $\mathbf{W}_u$ for control effort of the form

$$\mathbf{W}_u(\theta) = \Theta^{-1} \quad (15)$$

with diagonal and invertible Θ is proposed. This filter $\mathbf{W}_u$ individually increases the gain of the control effort of a saturated actuator without interference to the other actuators. The advantage of using Θ^{-1} becomes clear if the resulting controller $\mathbf{K}_{W_u}$

$$\begin{bmatrix} \dot{\mathbf{x}}_k \\ \mathbf{u} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_k & \mathbf{B}_k \\ \Theta \mathbf{C}_k & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x}_k \\ \mathbf{y} \end{bmatrix} \quad (16)$$

is analyzed. Compared to the controller $\mathbf{K}$ shown in (5), the new controller $\mathbf{K}_{W_u}$ linearly reduces the control action of saturated actuators according to the saturation matrix Θ . The details of the controller reconstruction, yielding this result, can be found in Wu et al. (2000).

6. CONTROLLER TUNING

The development of controllers with optimal trade-off between ride comfort and road-holding demands an additional pareto optimization loop as suggested in Unger et al. (2013) and Do et al. (2012). This optimization loop is crucial firstly because the induced L_2 -norm minimized during controller synthesis is not equal to the criteria, which quantitatively describe road-holding J_{rh} and ride comfort J_c :

$$J_{rh} = \sqrt{\frac{1}{N_{wl}} \sum_{k=1}^{N_{wl}} F_{wl,k}}, \quad (17)$$

with N_{wl} the number of samples of the stacked wheel loads F_{wl} , and the combination of the frequency weighted

vibration dose values of heave acceleration $vdv_{b,h}$ and angular pitch acceleration $vdv_{b,p}$

$$J_c = \sqrt{(k_1 vdv_{b,h})^2 + (k_2 vdv_{b,p})^2}. \quad (18)$$

The scalings $k_1 = 1$ and $k_2 = 2.34$, and the frequency weighting are taken from Hennecke (1995). Secondly, because the selected performance outputs, namely, body and wheel velocities, differ from the signals generally employed to characterized ride comfort and road-holding, see (17) and (18).

The pareto optimization is implemented using the non-linear FVM of ROMO identified in Fleps-Dezasse and Tobolar (2015) and a genetic algorithm from the DLR Matlab MOPS toolbox, see Joos et al. (2002). The controller is simulated with the nonlinear FVM in closed-loop and the criteria are evaluated from the simulation outputs. In this simulation, all sensor signals are corrupted with Gaussian noise and additionally, the acceleration signals are corrupted with a small bias. The tuning parameters of the observer synthesis w_g and $\mathbf{W}_{\Delta e_x}$, and of the controller synthesis w_w , $w_{b,h}$, $w_{b,r}$ and $w_{b,p}$ are optimized in one iteration step to achieve an optimal DF output-feedback controller. All further tuning parameters, like the sensor noise weight $\mathbf{W}_n$ and the bandwidths of the first-order low-pass filters in (13), are determined in advance. In particular, the bandwidths of the first-order filters are defined such that these filters roll-off before the invariant point in the frequency response of the body signals (Savaresi et al. (2010)).

In order to make the controller robust against different road disturbances, two road excitations are simultaneously considered in a multi-excitation approach:

- (1) a stochastic road excitation, and
- (2) an obstacle event (roughly corresponding to an impulse excitation).

7. SIMULATION RESULTS

The simulation investigation gives a comparison between the proposed full-vehicle LPV controller called *LPV FVM*, a full-vehicle Skyhook/Groundhook (*SH/GH*) controller and passive suspension configurations with constant $u_d \in [0.1 \ 0.3]$. Furthermore, the investigation shows results of two quarter-vehicle LPV controller, called *LPV QVM 1* and *LPV QVM 2*. The *LPV QVM 1* controller adopts the DF scheme and the performance signals and weights from the *LPV FVM*, while the *LPV QVM 2* controller follows a general output-feedback controller design with the performance signals and weights from Do et al. (2010). Additionally, the latter controller assumes zero tire damping and consequently the road profile height is used as disturbance input. The two quarter-vehicle LPV controllers are incorporated in the simulation investigation to firstly assess the benefit of the full-vehicle model and to secondly show the suitability of the DF design scheme. All controllers are tuned with the pareto optimization described in section 6 and their results are illustrated in conflict diagrams as introduced in Fig. 1. For visualization, the comfort and road-holding criteria are normalized to the highlighted passive configuration (red circle). Fig. 8 shows the results after the multi-excitation pareto optimization.

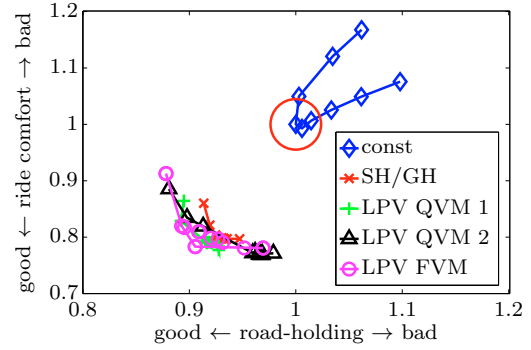


Fig. 8. Result of the multi-excitation pareto optimization

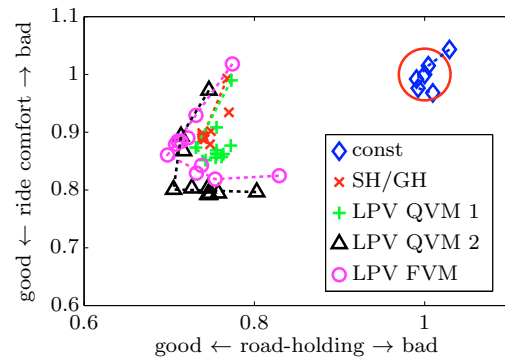


Fig. 9. Conflict diagram; excitation obstacle event

All four controllers mitigate the trade-off between ride comfort and road-holding compared to the passive suspensions. The three LPV controllers achieve almost the same ride comfort optimum and are slightly better than the SH/GH controller. With respect to road-holding, the results are very similar, but the *LPV FVM* and *LPV QVM 2* controller are able to improve road-holding by about 5% compare to the SH/GH controller.

Fig. 9 to Fig. 11 depict the conflict diagrams after separately evaluating the controllers in simulation subject to a heave sweep excitation, the obstacle event and the bad country road excitation. In these diagrams the respective best controllers are connected with dashed lines to simplify the interpretation. These dashed lines do not represent pareto-fronts like the solid lines in Fig. 8. In Fig. 9 it is found, that the *LPV QVM 2* controller achieves the best result for the obstacle excitation, especially the ride comfort is clearly improved. In contrast the *LPV QVM 2* performs worse than the other controllers in Fig. 10 and Fig. 11. This indicates that the *LPV QVM 2* controller specifically adapts to the obstacle excitation during the pareto optimization and as a drawback its performance deteriorates for other excitations. The *LPV FVM* controller achieves the best road-holding cost for all three excitations even though the differences are small. The ride comfort of the LPV FVM compared to the *LPV QVM 1* and the *SH/GH* controller are almost the same in Fig. 10 and Fig. 11, but a small improvement can be found for the obstacle excitation. The simulation results can be summarized as follows:

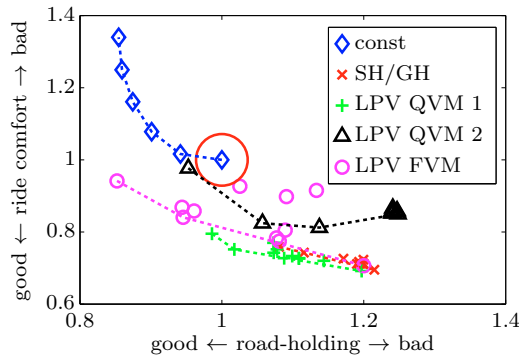


Fig. 10. Conflict diagram; excitation: heave sweep

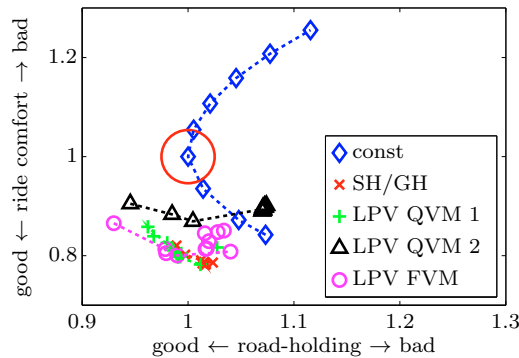


Fig. 11. Conflict diagram; excitation: bad country road

- (1) the proposed DF output-feedback controller design leads to controllers with similar performance than the reference output-feedback controller, and
- (2) in contrast to the expectation the LPV FVM controller yields only a small improvement of the trade-off between ride comfort and road-holding, compared to the LPV QVM controllers.

8. CONCLUSION

In this work a disturbance-feedforward output-feedback LPV controller design for the full-vehicle vertical dynamics has been presented. The controller explicitly considers the actuator saturation limits of the semi-active damper utilizing saturation indicators. In the LPV design a new saturation indicator dependent weighing scheme for a full-vehicle model based controller synthesis is proposed. This weighing scheme yields a controller which linearly reduces the respective controller output of saturated actuators. Besides the design targets ride comfort and road-holding, the computational effort during controller synthesis and controller code execution on real-time hardware is considered in the weighing scheme. A simulation investigation has shown that the proposed DF LPV output-feedback controller achieves a similar ride comfort and road-holding performance than a reference output-feedback controller while the problem complexity is drastically reduced. In the simulation study it could further be shown that a full-vehicle controller designed according to the presented weighing scheme provides a better trade-off between ride comfort and road-holding compared to the quarter-vehicle LPV controllers and the Skyhook/Groundhook controller

even though the difference has not been as large as expected. In the future, it is planned to investigate the LPV full-vehicle controller in more detail and to evaluate its performance on real-time hardware in the ROMO.

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